Collaborative Filtering on a Budget

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Outline

1. Main contribution
2. Algorithm details
3. Hashing
4. Modified algorithm with Hashing
5. Results

Main contribution

Use hashing so that required memory is *independent* of number of users and number of user/item features.

Notation

- Let Y be the rating matrix. Y_{ij} is the rating of the i^{th} user for the j^{th} item.
- Let F be the matrix of the predicted ratings.
- Then- $F_{ij} = \langle U_i, M_j \rangle$ and $U_i, M_j \epsilon \mathbb{R}^d$

Regularization

- Standard: $\Omega[U, M] = \frac{1}{2}\left[||U||^2_{Frob} + ||M||^2_{Frob}\right]$
- Proposed: $\Omega[U, M] = \frac{1}{2}\left[\sum_i n_i.||U_i||^2 + \sum_j m_j.||M_j||^2\right]$
- Minimize: $R[U, M] = L[UM^T, Y] + \lambda\Omega[U, M]$

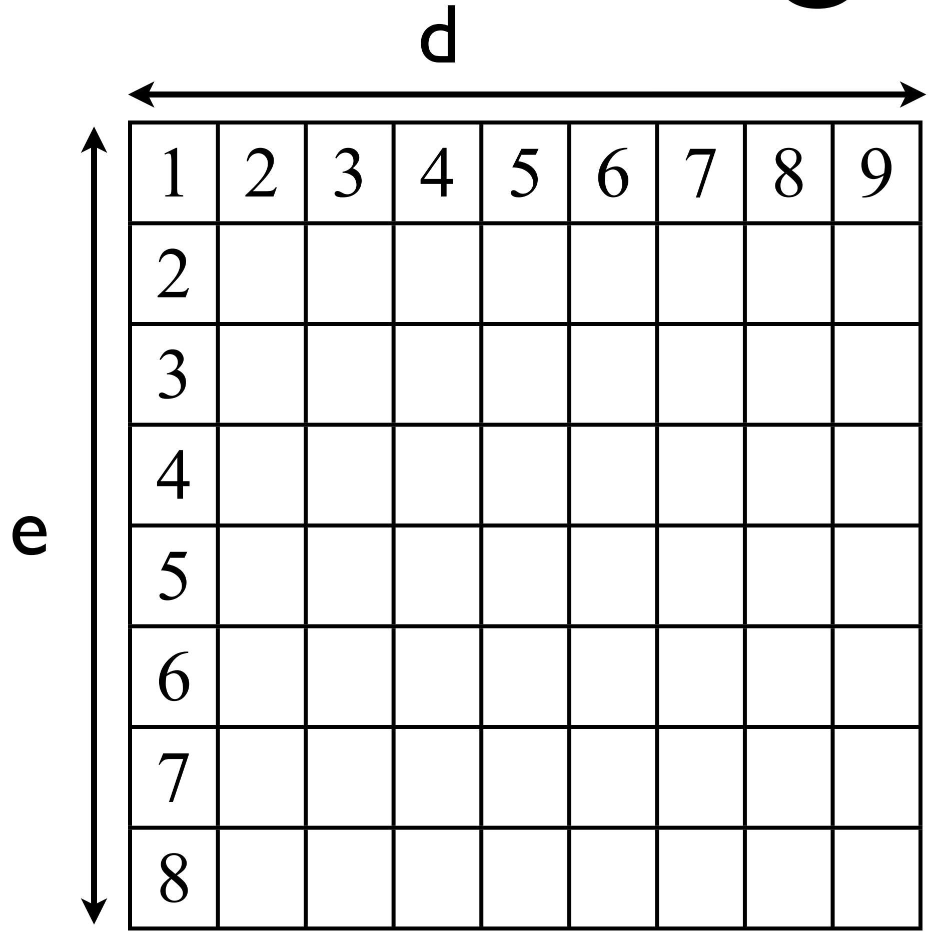
n_i and m_j are scaling factors that depend on the number of ratings

The algorithm

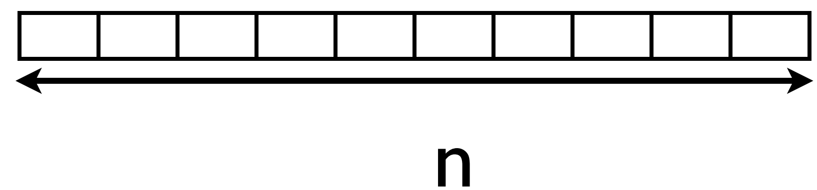
Algorithm 1 Matrix Factorization

Input Y, d
Initialize $U \in \mathbb{R}^{n \times d}$ and $M \in \mathbb{R}^{m \times d}$ with small random values.
Set $t = t_0$
while (i, j) in observations Y **do**
  $\eta \longleftarrow \frac{1}{\sqrt{t}}$ and $t \longleftarrow t + 1$
  $F_{ij} := \langle U_i, M_j \rangle$
  $U_i \longleftarrow (1 - \eta\lambda)U_i - \eta M_j \partial_{F_{ij}} l(F_{ij}, Y_{ij})$
  $M_j \longleftarrow (1 - \eta\lambda)M_j - \eta U_i \partial_{F_{ij}} l(F_{ij}, Y_{ij})$
end while
Output U, M

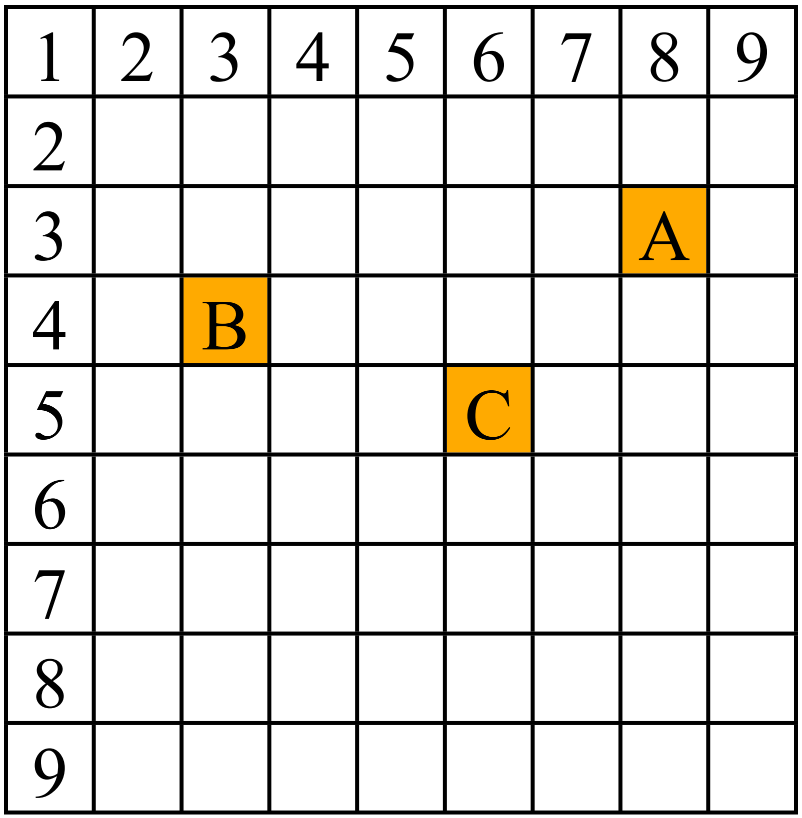
Hashing- The idea

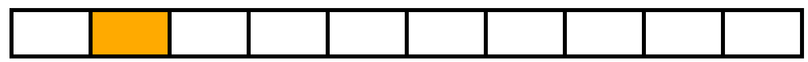


A hash function h, maps h (1 to e, 1 to d) to 1 to n
Say h(3,8) = h (4,3) = h (5,6) =2



Hashing- The idea





$$u_2 = \sum \left(A \cdot \sigma(3,8) + B \cdot \sigma(4,3) + C \cdot \sigma(5,6)\right)$$

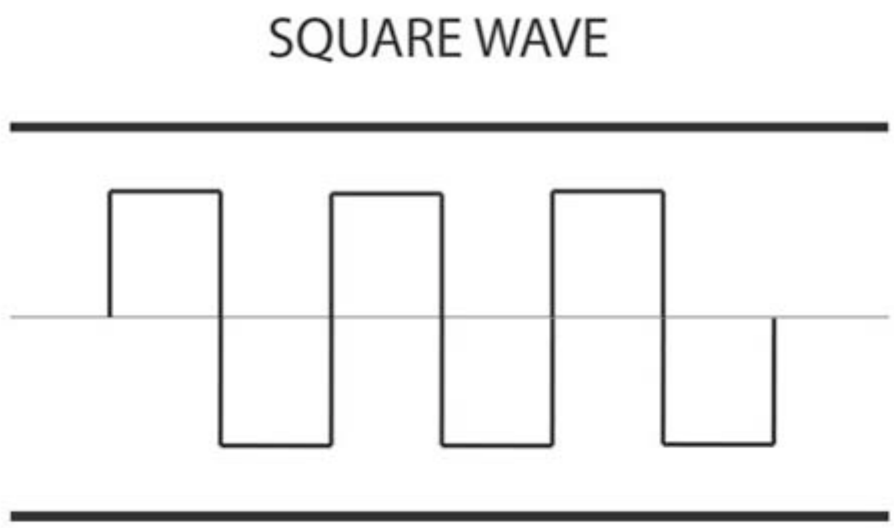
σ is the Rademacher function

Reconstructing: $A = u_2 \cdot \sigma(3,8)$

Works only when a small number of matrix values are significant

Rademacher function

- Also known as the square wave function! Looks like:



Mathematically...

Hashing:

$$u_i := \sum_{(j,k):h(j,k)=i} U_{jk}\sigma(j,k) \text{ and}$$
$$m_i := \sum_{(j,k):h'(j,k)=i} M_{jk}\sigma'(j,k)$$

Reconstruction:

$$\tilde{U}_{ij} := u_{h(i,j)}\sigma(i,j) \text{ and } \tilde{M}_{ij} := m_{h'(i,j)}\sigma'(i,j).$$

This allows us to reconstruct F_{ij} via

$$\tilde{F}_{ik} = \sum_j u_{h(i,j)} m_{h'(k,j)} \sigma(i,j)\sigma'(k,j).$$

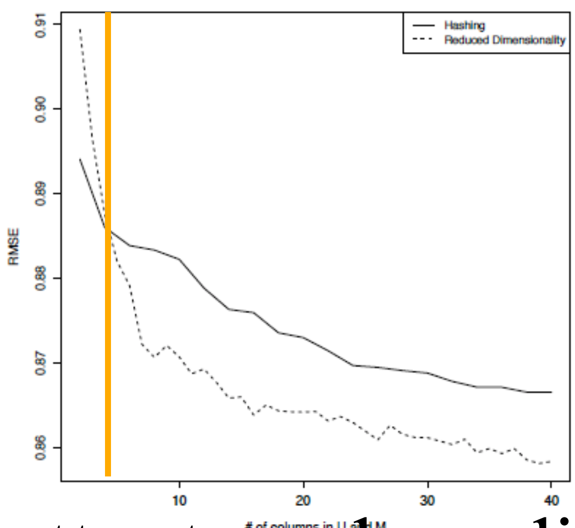
Modified algorithm with hashing

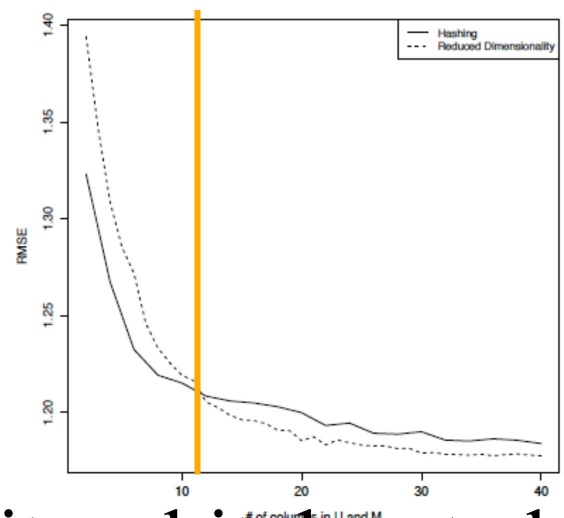
Algorithm 2 Compressed Matrix Factorization

Input Y,
Initialize $w \in \mathbb{R}^N$ with small random values.
Set $t = t_0$
while (i, k) in observations Y **do**
  $\eta \longleftarrow \frac{1}{\sqrt{t}}$ and $t \longleftarrow t + 1$
  $F_{ik} := \sum_j \sigma(i, j)\sigma'(k, j) w_{h(i,j)} w_{h'(k,j)}$
  $\gamma := \eta \partial_{F_{ik}} l(F_{ik}, Y_{ik})$
  $\mu := (1 - \eta\lambda)$
  for $j = 1$ **to** d **do**
    $w_{h(i,j)} \leftarrow \mu w_{h(i,j)} - \gamma \sigma(i, j)\sigma'(k, j) w_{h'(k,j)}$
    $w_{h'(k,j)} \leftarrow \mu w_{h'(k,j)} - \gamma \sigma(k, j)' \sigma(i, j) w_{h(i,j)}$
  end for
end while
Output w

Results

Squared	ϵ-insensitive	Huber
	MovieLens	
$\mathbf{0.857} \pm 0.006$	0.859 ± 0.004	$\mathbf{0.857} \pm 0.004$
	EachMovie	
1.177 ± 0.003	$\mathbf{1.161} \pm 0.007$	1.180 ± 0.005





Better to reduce dimensionality a bit than to hash. Use Hashing only when dimensionality needs to be reduced substantially.

Thank You!